Conservation of Angular Momentum

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Here, we know that $\vec{L} = \sum_i (\vec{r}_i \times \vec{p}_i)$

$$\frac{d\vec{L}}{dt} = ?$$

$$\Rightarrow \frac{d}{dt}\left[\sum_i \vec{r}_i \times \vec{p}_i\right] = \sum_i \left[\frac{d\vec{r}_i}{dt} \times \vec{p}_i + \vec{r}_i \times \frac{d\vec{p}_i}{dt}\right]$$

$\frac{d\vec{r}_i}{dt} \rightarrow \vec{u}_i$

$$= \sum_i \left[\underbrace{\vec{u}_i \times m_i \vec{u}_i}_{\to 0} + \vec{r}_i \times \frac{d\vec{p}_i}{dt}\right]$$

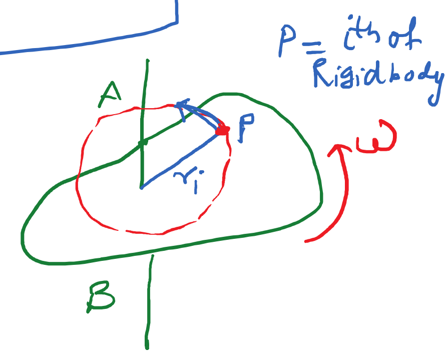
$$= \sum_i \left[0 + \vec{r}_i \times \vec{F}_i\right]$$

$$\boxed{\frac{d\vec{L}}{dt} = \sum_i \left[\vec{r}_i \times \vec{F}_i\right] \Rightarrow \vec{\tau}}$$

$ Second Method :

$\boxed{l = m u r}$

$\boxed{u_i = r_i \omega}$



$$l_i = m_i u_i r_i$$

$$= m_i r_i \omega r_i$$

$$\boxed{l_i = m_i r_i^2 \omega}$$

$$L = \sum_i l_i = \left(\sum_i m_i r_i^2\right)\omega \Rightarrow \boxed{L = I\omega}$$

$$\Rightarrow \frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} \qquad \left[\frac{d\omega}{dt} = \alpha\right]$$

$$\boxed{\frac{d\vec{L}}{dt} = I\alpha = \tau_{Ext}}$$

$$\boxed{\frac{dL}{dt} = \tau_{Ext}}$$

If the torque is zero then the angular momentum will be conserved.

This is known as principle of Conservation of Angular Momentum.

Rotational Kinetic Energy:

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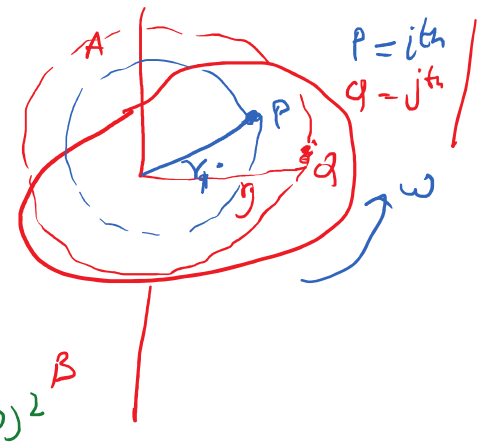


⇒ $v_i = r_i\omega$ & $v_j = r_j\omega$

for ith particle   for jth particle

Kinetic Energy $K = \frac{1}{2}mv^2$

P particle = $K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i (r_i \omega)^2$

$= \frac{1}{2} m_i r_i^2 \omega^2$

$$\Rightarrow K = \sum_i K_i = \sum_i \cancel{\frac{1}{2} m_i r_i^2}\, \omega^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$\rightarrow I$

$$\boxed{K = \frac{1}{2} I \omega^2}$$

$ Moment of Inertia: —

Moment of inertia of a system about a line —

$$\boxed{I = \sum_i m_i r_i^2}$$

⇒ m_i = mass of ith particle

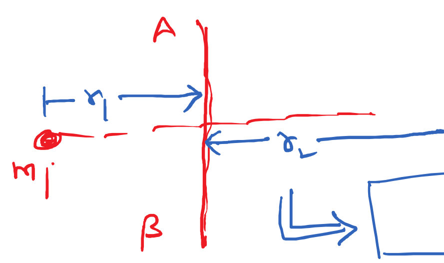
r_i = distance of ith particle from Axis

i = total No. of Particles $\sum_{i=0}^{N}$

$$\Rightarrow I = \sum_5 m_i r_i^2$$

$$\boxed{I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + m_5 r_5^2}$$

$\boxed{NP1}$



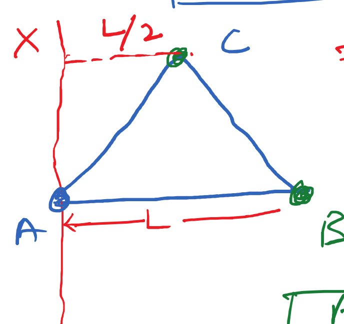
?

$\boxed{m_1 r^2 + m_2 r_2^2}$

$\boxed{NP2}$

Solution.

$$\Rightarrow \boxed{I = \sum_i m_i r_i^2}$$



⇒ mass of Each particle

$I = ?$

No. of particle in this system = 3

$\sum m_i r_i^2$

A → 1

B → 2

No. of particle in this system = 3

$$I = \sum_{3} m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$A \rightarrow 1$
$B \rightarrow 2$
$C \rightarrow 3$

$$\boxed{m_1 = m_2 = m_3 = m}$$

distance of particle A from Ax $= 0 = r_1$
" " " B " " $= L = r_2$
" " " C " " $= L/2 = r_3$

$$I = m(0)^2 + m(L)^2 + m\left(\frac{L}{2}\right)^2$$

$$= 0 + mL^2 + \frac{mL^2}{4}$$

$$\boxed{I = \frac{5mL^2}{4}}$$

Continuous Mass distributions:—

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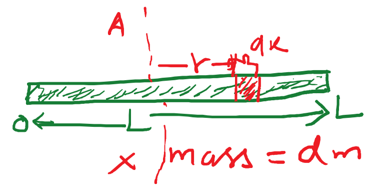
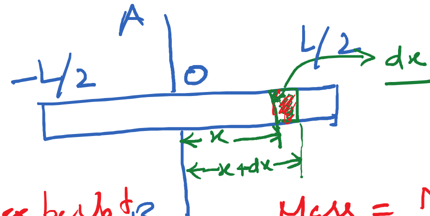
$$\boxed{I = \int r^2 dm}$$



$ Uniform rod about a ⊥ bisector:

$$\boxed{dm = \left(\frac{M}{L}\right)dx}$$



⇒ Mass of this element = (width) × Mass per unit length

Mass = M
Length = L
⇒ Mass per Unit length = M/L

$$\boxed{dm = \left(\frac{M}{L}\right)dx} \qquad \boxed{r = x}$$

$$I = \int x^2 \left(\frac{M}{L}\right) dx$$

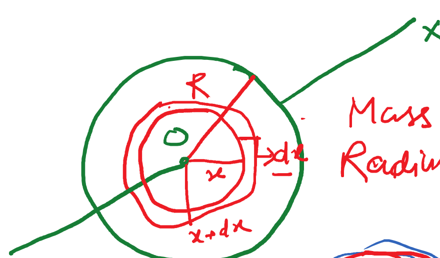
$$= \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx$$

$$= \frac{M}{L} \left[\frac{x^3}{3}\right]_{-L/2}^{L/2}$$

$$\boxed{I = \frac{ML^2}{12}}$$

$ Uniform Circular Plate:—

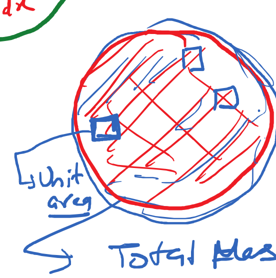
$$\boxed{I = \int r^2 dm}$$



Mass = M
Radius = R

dm = Mass of the ring we have chosen

⇒ Mass per Unit area $= \left(\frac{M}{\pi R^2}\right)$

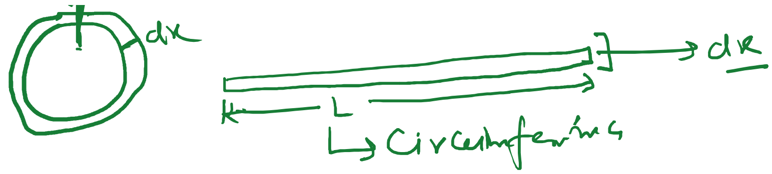


Total Mass = M

Area = πR^2

⇒ Total Mass / ... $= \left(\frac{M}{\pi R^2}\right)$

Area of the Ring:— $(\underbrace{2\pi x}_{\text{Length}})(\underbrace{dx}_{\text{width}})$



Area $= \pi R^2$

$\Rightarrow \frac{\text{Total Mass}}{\text{Total Area}} = \left(\frac{M}{\pi R^2}\right)$

Area of Rectangle = Length × Breadth

$= 2\pi x\, dx$

Mass of Ring = (Mass per Unit area) × area

$$dm = 2\cancel{\pi} x\, dx \cdot \frac{M}{\cancel{\pi} R^2}$$

$$r = x$$

$$I = \int \left(\frac{2M}{R^2}\right) x\, dx \cdot x^2$$

$$= \frac{2M}{R^2} \int_0^R x^3\, dx$$

$$\boxed{I = \frac{MR^2}{2}}$$